|  |
| --- |
| **Precalculus** |

**Unique Slopes**

|  |
| --- |
| Horizontal Lines = slope equals zero |
| Vertical lines = slope is undefined |

**Line Relationships**

|  |
| --- |
| Parallel Lines = always have an equal slope w/ each other |
| Perpendicular = always have the negative reciprocal slopes of each other |

**Function Notation Options**

|  |
| --- |
| \* F(x) = y  \* y is a function of x  \* dependent is a function of independent  \* units are y per x or y/x |

**X and Y**

|  |
| --- |
| X = independent  Y = dependent |
| X = domain  Y = range |
| X = input  Y = output |

**Linear Equations**

|  |
| --- |
| Standard Form  (Ax)+(By) = (C)  \* A,B,&C are constants |
| Point Slope Form  (y-yo) = m(x-xo)  \* (xo,yo) is a point on line  \* m = ave. rate of change |
| Slope Intercept Form  Y = mx+b  \* b = vertical intercept  \* m = ave. rate of change |
| If an equation can be put into one of the forms above, it is a linear equation |

**Quadratic Equations**

|  |
| --- |
| Standard Form  Y = (ax^2)+(bx)+c |
| Factored Form  Y = a(x-r)(x-s) |
| Vertex Form  Y=a((x-h)^2)+k |

**Two Operations to Avoid in the Domain**

|  |
| --- |
| 1. Avoid division by 0 2. Taking the even route of a negative number |

**Concavity**

|  |
| --- |
| How the average rate of change increases/decreases |

**Concave Up \_\_ =)**

|  |  |
| --- | --- |
| **Concave Up Decreasing** | **Concave Up Increasing** |
| \*Average rate of change increasing | \*Average rate of change increasing |
| \*Negative ave. rate of change | \*Positive ave. rate of change |
| concave up decreasing | concave up increasing |

**Concave Down \_\_ =(**

|  |  |
| --- | --- |
| **Concave Down Decreasing** | **Concave Down Increasing** |
| \*Average rate of change decreasing | \*Average rate of change decreasing |
| \*Negative ave. rate of change | \*Positive ave. rate of change |
| concave down decreasing | concave down increasing |

**Quadratic Equations**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Standard** | **Vertex** | **Factored** |
| **a** | **x** | **x** | **x** |
| **b** | **x** |  |  |
| **c** | **x** |  |  |
| **r** |  |  | **x** |
| **s** |  |  | **x** |
| **h** |  | **x** |  |
| **k** |  | **x** |  |

**Even Functions**

|  |
| --- |
| \* If f(-x) = f(x)  \* Graph of f is symmetric about the vertical axis (y-axis) |

**Odd Functions**

|  |
| --- |
| \* If f(-x) = -f(x)  \* Graph of f is symmetric about the origin (0,0) |

**Calculating Slope Between Two Points (m)**

|  |
| --- |
| \* ΔY/ ΔX  \* Δ = change in  \*m = (Y2 - y1)/(x2-x1)  \*Same as average rate of change |

**Quadratic Variables**

|  |  |
| --- | --- |
| a | \*If a>0, then parabola opens up \_&\_ k is min  \*If a<0, parabola opens down \_&\_ k is maximum |
| b | \*Moves parabola left and right |
| c | \*y-intercept |
| r/s | \*r ≤ s  \* r and s are x-intercepts |
| h/k | \* vertex = (h, k) |
| ax^2 | Quadratic term |
| bx | Linear term |
| c | Constant term |
| a | Leading coefficient |

**Law of Exponents**

|  |
| --- |
| 1. (A^m)\*(A^n) = (A^(m+n)) |
| 1. (A^m)^n) = (A^(m\*n)) |
| 1. ((A^m)/(a^n)) = (A^(m-n) |
| 1. ((A\*B)^n) = ((A^n)\*(B^n)) |
| 1. ((A/B)^n) = ((A^n)/(B^n)) |
| 1. ((A/B)^-n) = ((B/A)^n) |
| 1. ((A^-n)/(B^-m)) = ((B^m)/(A^n)) |

**Graphing**

|  |  |
| --- | --- |
| **Included** | **Not Included** |
| ≤ , ≥ | < , > |
| • | ○ |
| Solid line | Dashed line |
| [ ] | ( ) |
| { } Part of a piece-wise function, when only specific values are included | |

**Inflection Point**

|  |
| --- |
| Where concavity changes |

**Inverse Function**

|  |
| --- |
| Switch the x & y values of each point on a function |
| \* (F o F ^-1)(x) = x  \*(F^-1 o F)(x) = x |
| \* mirror each other across the x = y line |
| inverse function |

**Euler’s Number**

|  |
| --- |
| e = 2.71828 |

**Horizontal Line Test**

|  |
| --- |
| \* See if function’s inverse can be a function |

**Quadrants**

|  |
| --- |
| quadrants |

**Quadratic Equation**

|  |
| --- |
| quad equation |

**Exponential Functions**

|  |  |
| --- | --- |
| \*y = ab^x | \*y=ab^t |
| \* a = initial value | |
| \* b = growth factor | |
| \* b = 1 + r  \* r = b - 1 | |
| \* r = growth rate | |
| \* x (or t) = time | |
| \* a cannot equal 0 | |
| \* b > 1 = exponential growth  \* b < 1 = exponential decay | |

**Order of Function Transformations**

|  |  |  |
| --- | --- | --- |
| **-A f(-B(x-h)) + k** | | |
| 1 | B | Horizontal stretch/ compression |
| 2 | - | Reflection over y-axis |
| 3 | h | Horizontal shifting |
| 4 | A | Vertical stretch or compression |
| 5 | - | Reflection over x-axis |
| 6 | k | Vertical shifting |
| B > 1 = shrink  0 < B < 1 = stretch | | |
| A > 1 = stretch  0 < A < 1 = shrink | | |

**Graphing Periodic Transformations**

|  |  |
| --- | --- |
| A | Amplitude |
| k | midline |

**Find Period**

|  |  |
| --- | --- |
| Cos & Sin | 2π/ B |
| Tangent | Π/ B |

**Logs**

|  |  |  |  |
| --- | --- | --- | --- |
| Log or log base 10 | | | |
| Ln or log base e /natural log | | | |
| Y = 10^x | | = | log(y)=x |
| Y = e^x | | = | ln(y)=x |
| Log 1 = 0 | | = | (10^0) = 1 |
| Log 10 = 1 | | = | (10^1) = 10 |
| Log100=2 | | = | (10^2)=100 |
| If N>0 | 10^(logN) = N | | |
| For any N | Log(10^N) = N | | |

**Log Rules**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Ln(x) | | = | | logeX | |
| Ln(xy) | | = | | Ln(x)+Ln(y) | |
| Ln(x/y) | | = | | Ln(x)-Ln(y) | |
| Ln(1/x) | | = | | -Ln(x) | |
| Lnx^y | | = | | yLnx | |
| Ln(e)=1 | Ln(1)=0 | | | | (e^1)=e |
| (e^0)=1 | | | 10^(log(t))=t | | |
| e^(Lnx)= x | | | Ln(e^x) = x | | |

**Change of Base Theorem**

|  |
| --- |
| LogBX = Log10X / Log10B |

**Log Change**

|  |  |  |
| --- | --- | --- |
| Y = 10^x | = | Log(y) = x |
| Y = 10^x | inverse | Log(x) = y |

**Log vs Ln**

|  |
| --- |
| Ln vs Log 2 |
| ln vs log 3 |

**Vertical Asymptote Annotation**

|  |
| --- |
| (A^+ ) = approaching vertical asymptote from right hand side |
| (A^- ) = approaching vertical asymptote from left hand side |

**Circumference of a Circle**

|  |
| --- |
| C = 2πr |

**Comparing Exponential Functions**

|  |
| --- |
| **Non-Continuous**  Y = a(b^t) |
| **Continuous**  Y = a(e^rt) |
| A = vertical intercept  B = controls steepness |

**Compound Interest**

|  |  |
| --- | --- |
| P((1+r/n)^nt) | |
| t | Time(usually in years) |
| ht | Number of periods |
| r | Annual growth rate |
| P | Initial investment |
| r/n | Growth rate (per compounding period) |

**Continuous Compounding**

|  |  |
| --- | --- |
| P(e^rt) | |
| P | Principal amount |
| e | Euler’s number |
| r | Growth/decay rate |
| t | time |
| \* positive r = exponential growth  \* negative r = exponential decay | |

**Non-Continuous to Continuous**

|  |
| --- |
| Ln(b) = continuous rate |

**Reflections w/Exponential Functions**

|  |
| --- |
| Y = -2^(-t) |
| \*- in front of 2 = reflection over x-axis |
| \* - in front of t = reflection over y-axis |

**Phi and Theta**

|  |  |  |
| --- | --- | --- |
| θ | theta | Main angle |
| Φ | phi | Other angle |

**Phase Shift**

|  |
| --- |
| Φ = -Bh |
| Φ/2π = fraction of shifting |
| The fraction of the period shifting by 2π |

**Arc Length**

|  |
| --- |
| Arc Length = radians |
| S = rθ |
| Arc Length = radius\*angle (degrees) |

**Conversion of Degrees to Radians & vice versa**

|  |  |  |
| --- | --- | --- |
| D/360  degrees | = | R/2π  radians |

**SOH - CAH - TOA**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| soh cah toa | | | | | |
| sinθ | = | | Opposite/ hypotenuse | | |
| cosθ | = | | Adjacent/ hypotenuse | | |
| tanθ | = | | Opposite/ Adjacent | | |
| circle | | | | | |
| sinθ | | O/H | | Y/1 | Y |
| cosθ | | A/H | | X/1 | x |
| If y = sinθ & x = cosθ then p = (cosθ, sinθ) | | | | | |

**Angle**

|  |
| --- |
| \*measured from initial side to terminal side  \*clockwise = negative  \*counterclockwise=positive  \*terminal side is the hypotenuse  \* initial side = x-axis |

**Equations**

|  |
| --- |
| \* (A^2) + (B^2) = (C^2) |
| \* (X^2) + (Y^2) = (R^2) |
| \* R = radius |
| \*((cosθ)^2) + ((sinθ)^2) = (radius^2) |

**Law of Sines**

|  |  |  |
| --- | --- | --- |
| Sin(A) /  a | Sin(B)/  b | Sin(C)/  c |
| A (uppercase) = angle | | |
| a (lowercase) = side length | | |

**Odd & Even Identities**

|  |  |
| --- | --- |
| Even | Odd |
| 1. Cos(-x) =cos(x) 2. sec | 1. Sine 2. Csc 3. Tan 4. Cot |

**Reciprocal Identities**

|  |  |  |
| --- | --- | --- |
| Cot(θ) | = | 1/tan(θ) |
| Tan(θ) | = | 1/cot(θ) |
| Csc(θ) | = | 1/sin(θ) |
| Sec(θ) | = | 1/cos(θ) |
| Cos(θ) | = | 1/sec(θ) |
| Sin(θ) | = | 1/Csc(θ) |

**Quotient Identities**

|  |  |  |
| --- | --- | --- |
| Tan(θ) | = | Sin(θ)/cos(θ) |
| Cot(θ) | = | Cos(θ)/sin(θ) |

**Pythagorean Identities**

|  |
| --- |
| (Cos^2(θ)) + (sin^2(θ)) = 1 |
| 1 + (tan^2(θ)) = (sec^2(θ)) |
| (Cot^2(θ)) + 1 = (csc^2(θ)) |

**Power Reducing Identities**

|  |
| --- |
| (cos^2(θ))= (1/2)(1+(cos2θ)) |
| (sin^2(θ))= (1/2)(1-(cos2θ)) |

**Periodic Functions**

|  |
| --- |
| **periodic function** |
| **Periodic Function** = Values repeat at regular intervals |
| **Amplitude** = Vertical distance from the midline to the max or min value |
| **Midline** = Horizontal line midway between the max and min values |
| **Period** = Width of shortest repeating part |

**Law of Cosines**

|  |
| --- |
| \*(a^2) = (b^2) + (c^2) - 2bc(cos(A)) |
| (b^2) = (a^2) + (c^2) - 2ac(cos(B)) |
| (c^2) = (a^2) + (b^2) - 2ab(cos(C)) |

**What Quadrants Reciprocal Identities Are In**

|  |  |
| --- | --- |
| ArcCos | ArcTan  ArcCos  ArcSin |
|  | ArcTan  ArcSin |

**Reciprocal Graphs**

|  |  |  |
| --- | --- | --- |
| ArcSin | ArcCos | ArcTan |
| arc sin | arc cos | arc tan |

**Cartesian / Rectangular Coordinates**

|  |
| --- |
| (x,y) |

**Polar Coordinates**

|  |
| --- |
| (r,θ) |

**Cartesian to Polar**

|  |
| --- |
| r\*(cos(θ)) = x |
| r\*(sin(θ)) = y |

**Polar to Cartesian**

|  |
| --- |
| (r^2) = (x^2) + (y^2) |
| tan(θ) = (y/x) |

**Where Quadratic Equations are Negative**

|  |  |
| --- | --- |
| Tangent  Cosine |  |
| Cosine  Sine | Tangent  Sine |

**Inverse vs Reciprocal**

|  |  |
| --- | --- |
| (Sin^-1(x)) | Inverse function |
| (sinx)^-1 | Reciprocal function (cot, csc) |

**Order of Dominance of Different Types of Functions**

|  |
| --- |
| 1. Exponential (highest) |
| 1. Power |
| 1. Logarithmic (lowest) |

**Directly Proportional**

|  |
| --- |
| Y = k(x^p) |

**Inversely Proportional**

|  |
| --- |
| Y = k/(x^p) |

**Double Angle Formulas**

|  |  |
| --- | --- |
| Sine | (sine2θ) = 2(cosθ)(sinθ) |
| Cos | (cos2θ) = (cos^2(θ)) - (sin^2(θ)) |
| Tan | (tan2θ) = 2tan(θ) /  (1-(tan^2(θ)) |

**Domain & Range of Reciprocal Functions**

|  |  |
| --- | --- |
| Arc Sine | |
| Domain:  [-1 , 1] | Range:  [-π/2, π/2] |
| Arc Cos | |
| Domain:  [-1, 1] | Range:  [0, π] |
| Arc Tan | |
| Domain:  (-∞, ∞) | Range:  (-π/2 , π/2) |

**Find Vertical Asymptotes of Tangent**

|  |
| --- |
| 1. Start with original asymptotes (-π/2 & π/2) |
| 1. Divide each asymptote by B |
| 1. Add H |

**Rational Functions**

|  |
| --- |
| rational functions |

**Long Term Behavior - Rational Functions**

|  |  |
| --- | --- |
| Larger power in numerator | Approaching either +/- ∞ |
| Larger power in denominator | Approaching 0 |
| Equal powers in denominator & numerator | Approaching leading coefficient |

**Find Hole in Rational Funct.**

|  |
| --- |
| The y-value of the hole can be found by plugging the x into the remaining/ reduced function |
| Hole means that the value is not in the domain of the function |

**Find Horizontal Asymptotes of Rational Functions**

|  |  |
| --- | --- |
| y = | (A1x^B1) + (x) + 2 |
| (A2x^B2) + (x) + 1 |
| 1. If B1 > B2 (highest exponent in numerator > highest exponent in denominator), then no horizontal asymptote | |
| 1. If B2 < B1, then horizontal asymptote at (A1/A2)   \* If there’s a separate number added to rational function then that raises the horizontal asymptote | |
| 1. If B1 = B2, then look @ (A1/A2) like when B2 < B1 | |

**Sum & Difference Formulas**

|  |  |  |  |
| --- | --- | --- | --- |
| sin(Φ)cos(θ) | sin(Φ)cos(θ) | sin(θ)sin(Φ) | sin(θ)sin(Φ) |
| + | - | - | + |
| sin(θ)cos(Φ) | sin(θ)cos(Φ) | cos(θ)cos(Φ) | cos(θ)cos(Φ) |
| = | = | = | = |
| Sin(θ + Φ) | Sin(θ - Φ) | cos(θ + Φ) | cos(θ -Φ) |

**What is a Function?**

|  |
| --- |
| Can have repeating y values but not repeating x values |

**Is the Inverse of a Function a Function?**

|  |
| --- |
| If there are no repeating y-values, then yes the inverse can be a function. |

**Unit Circle**

|  |  |  |  |
| --- | --- | --- | --- |
| Degre | Radian | Cos | Sin |
| **0** | **0** | **1** | **0** |
| 15 |  |  |  |
| 30 | π/6 | (√3)  /2 | 1/2 |
| 45 | π/4 | (√2)/2 | (√2)/2 |
| 60 | π/3 | 1/2 | (√3)/2 |
| 75 |  |  |  |
| **90** | **π/2** | **0** | **1** |
| 105 |  |  |  |
| 120 | 2π/3 | -1/2 | (√3)/2 |
| 135 | 3π/4 | -(√2) /2 | (√2)/2 |
| 150 | 5π/6 | -(√3) /2 | 1/2 |
| 165 |  |  |  |
| **180** | **π** | **-1** | **0** |
| 195 |  |  |  |
| 210 | 7π/6 | -(√3) /2 | -1/2 |
| 225 | 5π/4 | -(√2) /2 | -(√2) /2 |
| 240 | 4π/3 | -1/2 | -(√3)  /2 |
| 255 |  |  |  |
| **270** | **3π/2** | **0** | **-1** |
| 285 |  |  |  |
| 300 | 5π/3 | 1/2 | -(√3)  /2 |
| 315 | 7π/4 | (√2) /2 | -(√2) /2 |
| 330 | 11π/6 | (√3)/2 | -1/2 |
| 345 |  |  |  |
| **360** | **2π** | **1** | **0** |

**Types of Functions**

|  |  |
| --- | --- |
| Constant | Y = a |
| Linear | Y=mx+b |
| Exponential | Y=a(b^x) |
| Power | Y=a(x^b) |
| Quadratic | Ax^2+bx+c=0 |
| Rational | Y=A(x-2)(x+3) /(x-4)(x+3) |
| Polynomial | Y=An\*x^n+ An-1\*x^n-1+ An-2\*x^n-2 |
| Logarithmic | Y=A\*Ln(x)+b |
| Periodic | Y=sin(x) |

**Periodic Functions**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Asymptotes** | NA | NA | Θ = π/2 + πk | Θ = π/2 + πk | Θ = π + πk | Θ = π + πk |
| **Period** | 2π | 2π | π | 2π | 2π | π |
| **Amplitude** | 1 | 1 | NA | NA | NA | NA |
| **Starts at** | 1 | 0 | 0 | 1 | NA | NA |
| **Min** | 0 | 0 | NA | NA | NA | NA |
| **Max** | 1 | 1 | NA | NA | NA | NA |
| **Reciprocal**  **of** | secant | cosecant | cotangent | cosine | sine | tangent |
| **Short** | cos | sin | tan | sec | csc | cot |
| **Function** | **cosine** | **sine** | **tangent** | **secant** | **cosecant** | **cotangent** |

**Completing the Square**

**(Standard Form to Vertex Form)**

|  |
| --- |
| 1. Move C to the other side |
| 1. Factor out A if A doesn’t equal 0 |
| 1. Multiply B by (1/2) |
| 1. a. Square B (B^2) |
| 4.b.(only if A didn’t equal 0) Multiply the number from 4a by A (that was factored out in step 2) |
| 1. Add the number from 4a to C. (4b if you needed that step) |
| 1. Change the equation on the right of the equals sign to a perfect square with the value from step 3 |
| 1. Move the value from the left of the equals sign back to the right and you now have vertex form |
| 1. **Y = (2x^2) - (2x) - 12** |
| 1. **12 \_\_ = 2((x^2)-(x) + \_\_)** |
| 1. **(-1/1)\*(1/2) = (-1/2)** |
| 1. **a. (-1/2)\*(-1/2) = (1/4)** |
| **4.b. (1/4)\*(2/1) = (1/2)** |
| 1. **12+(1/2)=2((x^2)-x+(1/4))** |
| 1. **12.5 = 2[(x-(1/2))^2]** |
| 1. **Y = 2[(x-(1/2))^2] - 12.5** |

**Standard Form to Vertex Form Shortcut**

|  |
| --- |
| (ax^2)+(bx) + c = 0  A((x-h)^2) + k = 0 |
| H = (b/2a)  K = c - (b^2/4a) |

**Find Type of Function from a Table of Values**

|  |
| --- |
| **Linear** |
| Same average rate of change (slope) on every interval ((y2-y1)/(x2-x1)) |
| **Quadratic** |
| 1. Change in change of y has same # **added** at each step 2. Constant increase or decrease of x-values |
| **Exponential** |
| 1. Change in change of y has same # **multiplied** at each step 2. Constant change in x 3. Y-values constantly multiplied by the same number |
| **Periodic** |
| Repeats either y-values or x-values at regular intervals |